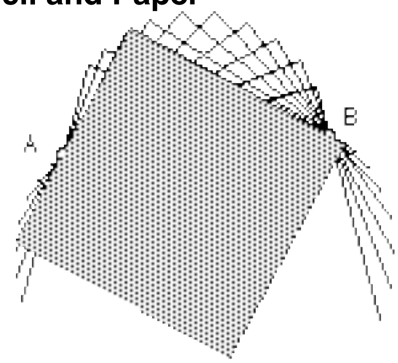
Exploration 4.1—Carpenter's Construction of a Circle

There is an interesting way to draw a circle using only a right-angle corner, called the *Carpenter's Construction of a circle*. (Carpenter's are fond of this method of drawing a circle because it does not require them to know where the center of the circle is located.) You can accomplish this construction with a carpenter's square, with an index card or the corner of a piece of paper, or with Sketchpad.

Investigation 1—Carpenter's Construction with Pencil and Paper

Construction



- Mark two points on a piece of paper.
- As shown at right, place an index card so that two adjacent sides of the card each touch one of the two points.
- Make a dot at the corner of the card with a pencil.
- Do this repeatedly until you have many dots. This is easier and quicker if you slide the card by placing guides at points *A* and *B*. A carpenter might use two nails in a piece of wood, but on paper you can have a partner hold two pencil points, one at *A* and one at *B*.

Conclusions

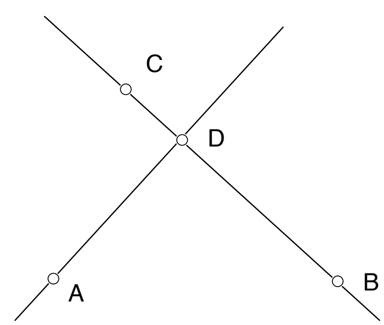
Q1. The corner of the card appears to move along the arc of a circle. Where does the center of this circle appear to be?

Q2. How can you use this method to draw a complete circle?

Investigation 2—Simulation of the Carpenter's Construction with Sketchpad

Now you will carry out the same construction with Sketchpad. You will need to construct the moving right-angle corner of the card.

Construction



- Start a sketch with a point *A* and a line *BC*. You can drag C to move the line through B to various positions. Think of C as the Controller.
- To form a right angle ADB, construct the perpendicular line to *BC* through *A* and let *D* be the intersection of the two lines.
- Now leave *A* and *B* fixed and move *C* about the plane. The angle *ADB* is a moving right angle, like the corner of the index card or of the carpenter's square.

Experiment

- Trace the path of point *D* as point *C* moves around in the plane. What does this trace appear to be?

- Conjecture the shape that you think the trace produces. Then construct this shape as an object with Sketchpad. (Do not use *D* in the construction, because you need to move *D*).

  If your conjecture is correct, then when you move *C* again, point *D* should move along the object, and its trace should lie on top of the object, even if you change the locations of points *A* and *B*.

Conclusions

Q1. What kind of shape is the trace? How did you construct it?

Q2. Why do you think the trace has this shape? (Give your best explanation now; reasons may become clearer after the next exploration.)

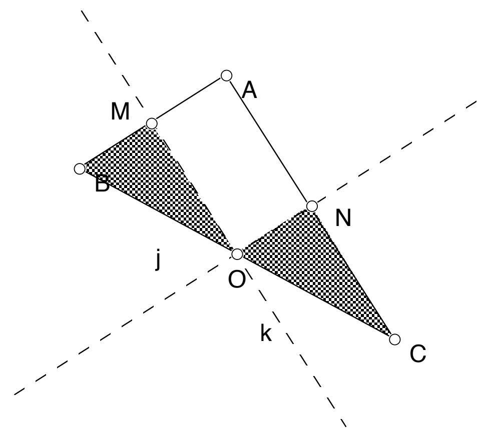
Exploration 4.2—Exploring Right Triangles

In seeking to explain how the Carpenter's Construction results in a circle, you will find out more about right angles, first by folding them physically with paper and then figuratively with Sketchpad.

Investigation 2—Cutting up Right Triangles with Sketchpad

Experiment

- Construct a right triangle ABC. Then construct the midpoints of the sides, labeled *M*, *N*, *O* as shown.
- Construct line *NO*. How is this line related to the sides *AB* and *AC*?
- Construct line *MO*. How is this line related to the sides *AB* and *AC*?
- What is angle NOM? Why?
- Construct the interiors of triangles *MBO* and *NOC* as shown. How are these triangles related to triangle ABC?
- Construct segment AO. This divides AMON into 2 more small triangles.



☞ Look for reasons to explain whether or not the four small triangles are congruent.

Consider what the relationships among the four small triangles say about the distances *OA*, *OB*, and *OC*.

- Construct the circle with center *O* through the point *A*.

☞ How is this circle apparently related to points *B* and *C*? Justify your statement.

Conclusions (summarizing your findings)

Q1. What kind of triangles are *AOB* and *AOC*?

Are triangles *AOB* and *AOC* congruent?

Explain the relationship among the lengths *OA*, *OB*, *OC* based on what you know about triangles *AOB* and *AOC*?

Q2. Are the four small triangles congruent? Give reasons for your answer.

Q3. What is the shape of the quadrilateral *AMON*? Justify your answer..

Q4. a. Explain why circle *OA* passes through *B* and *C*.

b. What special name does this circle have? (Refer, if necessary to Chapter 3.)

c. What term best describes the role of segment *BC* in relation to this circle?

d. In Chapter 3, the circumcenter of a triangle ABC was found to be the intersection of three perpendicular bisectors. Explain which lines are the perpendicular bisectors in this right triangle ABC and tell which point is the circumcenter.